

Return of the Boltzmann Brains ^{*}

Don N. Page [†]

Institute for Theoretical Physics
Department of Physics, University of Alberta
Room 238 CEB, 11322 – 89 Avenue
Edmonton, Alberta, Canada T6G 2G7

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Abstract

Linde in hep-th/0611043 shows that some (though not all) versions of the global (volume-weighted) description avoid the “Boltzmann brain” problem raised in hep-th/0610079 if the universe does not have a decay time less than 20 Gyr. Here I give an apparently natural version of the volume-weighted description in which the problem persists, highlighting the ambiguity of taking the ratios of infinite volumes that appear to arise from eternal inflation.

One of the most important challenges in theoretical cosmology today is the measure problem, the problem of giving theoretical predictions of the relative measures or probabilities for various types of observations. A quantum solution of this problem might be a specification of a quantum state of the universe that gives normalizable expectation values of positive operators associated with observations. (In [1, 2, 3] I have suggested that the fundamental observations are conscious perceptions, each set of which has its own positive quantum ‘awareness operator’ whose expectation value in the quantum state of the cosmos gives the measure of the corresponding set of conscious perceptions, but other theorists might prefer to interpret observations differently.)

A severe problem has arisen from the predictions of eternal inflation [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15], which seems to lead to an arbitrarily large universe, with an arbitrarily large number of observers, which makes it problematic how to calculate the probability of various observed features by taking the ratio of the numbers or measures of the corresponding infinite sets of observers [16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31]. This problem is highlighted by paradoxes, if the

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[†]Internet address: don@phys.ualberta.ca

universe lasts too long, of too many disordered observations being made by brains resulting from thermal fluctuations [32, 33, 34] (“Boltzmann brains” or BB’s) or from vacuum fluctuations [35, 36, 37] (“brief brains” or bb’s, though I shall often lump them all together as BB’s or BBs, not to be confused with the BBS or Blum-Blum-Shub pseudorandom number generator [38]). For example, it was found [36] that if the universe continues to expand exponentially at the rate given by present observations, then per comoving volume there would be an infinite expectation value of the 4-volume if the expected decay time is less than 13–19 Gyr (say 20 Gyr to give a conservative upper limit), leading to very tiny probabilities of observations as ordered as ours are.

Bousso and Freivogel [39] have recently argued that this paradox does not arise in the local description [40] (unless the decay time is much, much longer) and is evidence against the global description of the multiverse. On the other hand, Linde [41] has more recently given an example of how one may avoid the dreaded “invasion of Boltzmann brains” within his ‘standard’ volume-weighted distribution in the global description. Here I wish to point out a natural different regularization of the volume-weighted distribution in which the problem of the Boltzmann brains returns.

Linde’s solution in Section 8.3 of [41] has at least two de Sitter vacuum states, one (labeled by the subscript 1) with a small expansion rate H_1 consistent with our observations, and another (labeled by the subscript 2) with a larger expansion rate H_2 . Then as a function of the global cosmic proper time t , spacetime in state 2 will expand rapidly (asymptotically giving the main contribution to the volume growth of both types of regions) and will with some transition rate Γ_{21} convert to state 1 where both ordinary observers (OOs, “honest folk like ourselves” [41]) and Boltzmann or brief brains (BBs) can exist. Then even though each spacetime region in state 1 will expand indefinitely and produce far more BBs than OOs, at each time more spacetime region in state 1 is being produced by transitions from the more rapidly expanding spacetime in state 2, so that asymptotically at each large but fixed proper time, nearly all of the spacetime regions in state 1 will arise from recent transitions from state 2, rather than from the slower expansion of state 1. Thus at each time, most of the volume in state 1 will be early in its evolution, where OOs rather than BBs dominate. In particular, if $3(H_2 - H_1) \gg \Gamma_{1B}$, the rate of the BB production in state 1, then at each fixed late proper time, there will be far more ordinary observers than Boltzmann brains or brief brains from vacuum fluctuations.

For example, in [37] I made the estimate that the probability per 4-volume for a brief brain is $\Gamma_{1B} \sim e^{-10^{42}}$, where the upper exponent had already been predicted 3^3 years ago [42]. This rate is indeed much, much less than any reasonable $3(H_2 - H_1)$.

Linde’s solution is analogous to the following situation. Consider imaginary humans who have a ‘youthful’ phase of 100 years of life with frequent and mostly ordered observations, followed by a ‘senile’ phase of trillions of years of infrequent and mostly disordered observations. Assume that the trillions of years are sufficient to give more many total ‘senile’ disordered observations than ‘youthful’ ordered ob-

servations for each human. One might think that most observations would then be disordered, so that someone’s having an ordered observation (which would thus be very improbable under this scenario) could count as evidence against the theory giving this scenario. However, if the population growth rate of such humans is sufficiently high that at each time the number of youthful humans and their ordered observations outnumbers the senile humans and their disordered observations, Linde’s solution is that at each time the probability is higher that an observation would be ordered than that it would be disordered.

I agree that Linde’s solution is a possible way to regulate the infinity of observations that occur in a universe that expands forever, but it is not the only way, or, I might suggest, the most natural way. The problem arises from the fact that if the youthful humans or OOs are always at late times to outnumber the senile humans or BBs, the population of these fictitious humans, or the volume of the universe in the original example, must continue growing forever, producing an infinite number of both youthful and senile fictitious humans or of both OOs and BBs in cosmology. Then it is ambiguous how one takes the ratio, which is the fundamental problem with trying to solve the measure problem in theories with eternal inflation. Linde himself, in Section 8.2 of [41], gives a “pseudo-comoving volume-weighted distribution” which he shows does have the problem of BBs dominating if the decay rate of state 1 is not higher than the BB production rate.

For the analogue with the fictitious humans, an alternate way to regulate the infinities (which seems simpler and more natural to me) would be just to count the observations of each such human, regardless of when they occur. Then under the given scenario, the senile ones would dominate, giving a low probability for having an ordered observation.

Similarly, in cosmology with spacetime regions 1 which allow observations and spacetime regions 2 which for simplicity I shall assume do not allow observations, an alternate simple way to regulate the infinities would be just to take the observers in a single connected region of spacetime in state 1. If this region was born by a transition from state 2 and died with a transition back to state 2 (or to any other state), then since by hypothesis observers can only exist within regions 1, it would be most natural to regulate the infinity that arises from the infinite number of regions 1 that occur by calculating the ratios of measures for observations within a single such connected region 1. Then if regions 1 with asymptotically exponential growth rates H_1 agreeing with observations in our part of the universe have half-lives longer than about 20 billion years, nearly all observations made within it would be by BBs, which would give almost entirely disordered observations, so our ordered observations would be very unlikely according to this theory, counting as strong observational evidence against it.

One might still worry that even our connected region that is entirely in state 1 might be spatially infinite (e.g., if $k = 0$ or $k = -1$), giving an infinite number of both OOs and BBs. In this case one would indeed need some other regularization in addition to that above (focusing on only one connected region), such as using only a

finite comoving volume. However, if our region came from inflation in a finite space, such as a $k = +1$ universe, then we could still use the entire spatial volume, which at each finite time would be finite. If the universe does expand faster than its decay rate (e.g., if its half-life is greater than 20 Gyr), then the total future expectation value of its 4-volume would be infinite, leading to an infinite expected number of BBs, but there would only be a finite number of OOs, so one could still conclude that the relative probability of an OO was zero, and of the BBs, only a very tiny probability would be of ordered observations. Therefore, our ordered observations would be strong evidence against this theory with this natural way of doing the regularization (focusing on just one connected region 1).

Thus Linde's 'standard' regularization in Section 8.3 of [41] is only one way to get finite ratios from the infinitely many observers that appear to arise from eternal inflation, and besides Linde's alternative prescription in Section 8.2 of [41], I have shown above that an apparently natural way, still using volume weighting, returns the problem of the Boltzmann brains.

Of course, the fact that we have ordered observations and are almost certainly not Boltzmann brains is strong evidence against what I have here proposed as a natural way of using volume weighting in the global viewpoint (unless the universe really is decaying with a half-life less than 20 billion years [36], which also seems rather implausible [36, 39, 41]). So in comparison with the observations, and under the assumption that the universe is not decaying within 20 billion years, my proposal is definitely worse than Linde's 'standard' prescription.

I mainly wish to highlight my opinion that the 'standard' prescription, with its emphasis on the global cosmic time at the end of the rapid inflationary period at the beginning of our region 1, is not an unambiguously natural proposal. I am happy to praise it as a clever step forward toward our understanding of the severe measure problem in cosmology (whose solution must be part of our ultimate theory, as Linde has emphasized in Section 9 of [41]). However, I do not yet feel that it is sufficiently unique or natural to be *the* final solution. So the purpose of the present paper is not to be critical of the valiant attempts to solve the measure problem, or to propose a solution that I think is better, but to highlight my opinion that the problem persists and to stimulate more intelligent OOs to look for a natural solution that I myself don't begin to see how to solve.

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